Unit 1
Matrices And Determinants

EXERCISE 1.1

Q1. Find the order of the following matrices.

$A = \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 4 \end{bmatrix}$,

$D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$, $E = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$, $F = [2]$,

$G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$, $H = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$

Solution:

Order of Matrix:

The number of rows and columns in a matrix specifies its order.

Ans. (i) Matrix A has two rows and two columns so it's order = number of rows × number of columns = 2-by-2.

Ans. (ii) Matrix B has two rows and two columns so it's order = number of rows × number of columns = 2-by-2.

Ans. (iii) Matrix C has one row and two columns so it's order = number of rows × number of columns = 1-by-2.

Ans. (iv) Matrix D has three rows and one column so it's order = number of rows × number of columns = 3- by-1.

Ans. (v) Matrix E has three rows and two columns so it's order = number of rows × number of columns = 3-by-2.

Ans. (vi) Matrix F has only one row and one column so it's order = number of rows × number of columns = 1-by-1.

Ans. (vii) Matrix G has three rows and three columns so it's order = number of rows × number of columns = 3-by-3.

Ans. (viii) Matrix B has two rows and three columns so it's order = number of rows × number of columns = 2-by-3.

Q2. Which of the following matrices are equal?

$A = [3]$, $B = \begin{bmatrix} 3 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 4 \end{bmatrix}$,

$$D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}, \quad E = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}, \quad F = \begin{bmatrix} 2 \\ 6 \end{bmatrix},$$

$$G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}, \quad H = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}, \quad I = [3 \quad 3+2],$$

$$J = \begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$$

Solution:

Matrices are said to be equal if

(i) They are of same order,

(ii) Their corresponding entries are equal.

So, according to this definition

Ans. **(a)** Matrices A and C are equal $A = C$.

(b) Matrices B and I are equal $B = I$.

(c) Matrices E, H and J are equal $E = H = J$.

(d) Matrices F and G are equal $F = G$.

Q3. Find the values of a, b, c and d which satisfy the matrix equation

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

Solution:

As, $\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$

By comparing the corresponding elements

So, $a + c = 0$

$a = -c$ ----------- (i)

$a + 2b = -7$

$2b = -(a + 7)$ ----------- (ii)

$c - 1 = 3$

$c = 3 + 1$

$c = 4$ ----------- (iii)

By putting the value of "c" in equation (i), we will get

$a = -4$ ---------- (iv)

By putting the value of "a" in equation (ii), we will get

$2b = -(-4 + 7)$

$2b = -(3)$

$b = -(3/2)$

$b = -1.5$ --------- (v)

Similarly,

$4d - 6 = 2d$

$$4d - 2d = 6$$
$$2d = 6$$
$$d = 6/2$$
$$d = 3 \quad \text{---------- (vi)}$$

From equations (iii), (iv),(v) and (vi) we get

$$a = -4, b = -1.5, c = 4 \text{ and } d = 3$$

EXERCISE 1.2

Q1. From the following matrices, identify unit matrices, row matrices, column matrices and null matrices.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \end{bmatrix}, \quad F = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

Solution: **Matrix A** is a null matrix (because it's all entities are zero).

Matrix B is a row matrix (because it has only one row).

Matrix C is a column matrix (because it has only one column).

Matrix D is a diagonal matrix (because it's diagonal entities are 1).

Matrix E is a null matrix (because it's all entities are 0).

Matrix F is a column matrix (because it has only one column).

Q2. From the following matrices, identify

(a) **Square matrices** **(b)** **Rectangular matrices**
(c) **Row matrices** **(d)** **Column matrices**
(e) **Identity matrices** **(f)** **Null matrices**

(i) $\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **(v)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ **(vi)** $\begin{bmatrix} 3 & 10 & -1 \end{bmatrix}$

(vii) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ **(viii)** $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **(ix)** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$4d - 2d = 6$$
$$2d = 6$$
$$d = 6/2$$
$$d = 3 \qquad \text{---------- (vi)}$$

From equations (iii), (iv),(v) and (vi) we get

$$a = -4, b = -1.5, c = 4 \text{ and } d = 3$$

EXERCISE 1.2

Q1. From the following matrices, identify unit matrices, row matrices, column matrices and null matrices.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \end{bmatrix}, \quad F = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

Solution: **Matrix A** is a null matrix (because it's all entities are zero).

Matrix B is a row matrix (because it has only one row).

Matrix C is a column matrix (because it has only one column).

Matrix D is a diagonal matrix (because it's diagonal entities are 1).

Matrix E is a null matrix (because it's all entities are 0).

Matrix F is a column matrix (because it has only one column).

Q2. From the following matrices, identify

(a) Square matrices **(b) Rectangular matrices**

(c) Row matrices **(d) Column matrices**

(e) Identity matrices **(f) Null matrices**

(i) $\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **(v)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ **(vi)** $\begin{bmatrix} 3 & 10 & -1 \end{bmatrix}$

(vii) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ **(viii)** $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **(ix)** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Solution:

(a) (iii),(iv) and (viii) are square matrices because the number of rows are equal to number of columns.

(b) (i),(ii),(v),(vi),(vii),(ix) are rectangular matrices because their rows and columns are not equal.

(c) (vi) is a row matrix because it has only one row.

(d) (ii) and (vii) are column matrices because they have only one column.

(e) (iv) is a identity matrix as well because its diagonal elements are "1".

(f) (ix) is a null matrix because its each entity is zero.

Q3. From the following matrices, identify diagonal, scalar and unit (identity) matrices.

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$$

Solution: **Matrix A** is a scalar matrix (because its diagonal entities are same).

Solution: **Matrix B** is a diagonal matrix (because its diagonal entities are non-zero and non diagonal entities are zero).

Solution: **Matrix C** is a identity matrix (because its diagonal entities are 1).

Solution: **Matrix D** is a diagonal matrix (because its one diagonal entity is non-zero and non-diagonal entities are zero).

Solution: **Matrix E** is a scalar matrix (because its diagonal entities are same).

Q4. Find negative of matrices A, B, C, D and E when:

$$A = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix},$$

$$D = \begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$$

Solution:

Negative of a matrix is obtained by inverting (changing) the signs of all of its entities.

So,

(i) $-A = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ **(ii)** $-B = \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix}$

(iii) $-C = \begin{bmatrix} -2 & -6 \\ -3 & -2 \end{bmatrix}$ **(iv)** $-D = \begin{bmatrix} 3 & -2 \\ 4 & -5 \end{bmatrix}$

(v) $-E = \begin{bmatrix} -1 & 5 \\ -2 & -3 \end{bmatrix}$

Q5. Find the transpose of each of the following matrices:

$\mathbf{A} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$,

$\mathbf{D} = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$, $\mathbf{E} = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$, $\mathbf{F} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Solution:

Transpose of a matrix is obtained by converting all the columns of that matrix to the rows and all the rows to the columns.

So, according to the definition;

(i) $A^t = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix}$ **(ii)** $B^t = \begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$

(iii) $C^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \end{bmatrix}$ **(iv)** $D^t = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$

(v) $E^t = \begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix}$ **(vi)** $F^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

Q6. Verify that if $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ then

(i) $(A^t)^t = A$ **(ii)** $(B^t)^t = B$

Solution:

(i) To prove; $(A^t)^t = A$

Given $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

Taking transpose of A^t, we will get

$(A^t)^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = A$

Hence proved: $\mathbf{(A^t)^t = A}$

Solution: **(ii)** To prove; $(B^t)^t = B$

Given $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

Taking transpose of B^t, we will get

$$(B^t)^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = B$$

Hence proved: $(B^t)^t = B$

EXERCISE 1.3

Q1. Which of the following matrices are conformable for addition?

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix},\quad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix},\quad C = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix},$$

$$D = \begin{bmatrix} 2+1 \\ 3 \end{bmatrix}\quad E = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix},\quad F = \begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix}$$

Solution:

Matrices of same order are conformable for addition. So, according to this definition;

(i) Matrices A and E are conformable for addition (because both have order 2-by-2).

(ii) Matrices B and D are conformable for addition (because both have order 1-by-1).

(iii) Matrices C and F are conformable for addition (because both have order 3-by-2).

Q2. Find the additive inverse of following matrices.

$$A = \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix},\quad B = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix},\quad C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix},\quad E = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},\quad F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$$

Solution:

The additive inverse of a matrix is obtained by changing the sign of each entity. So, according to the definition;

(i) Additive inverse of A = - A = $\begin{bmatrix} -2 & -4 \\ 2 & -1 \end{bmatrix}$

(ii) Additive inverse of B = - B = $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$

(iii) Additive inverse of C = - C = $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$

Taking transpose of B^t, we will get

$$(B^t)^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = B$$

Hence proved: **$(B^t)^t = B$**

EXERCISE 1.3

Q1. Which of the following matrices are conformable for addition?

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix},\ B = \begin{bmatrix} 3 \\ 1 \end{bmatrix},\qquad C = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix},$$

$$D = \begin{bmatrix} 2+1 \\ 3 \end{bmatrix}\quad E = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix},\ F = \begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix}$$

Solution:

Matrices of same order are conformable for addition. So, according to this definition;

(i) Matrices A and E are conformable for addition (because both have order 2-by-2).

(ii) Matrices B and D are conformable for addition (because both have order 1-by-1).

(iii) Matrices C and F are conformable for addition (because both have order 3-by-2).

Q2. Find the additive inverse of following matrices.

$$A = \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix},\quad B = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix},\quad C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix},\ E = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},\qquad F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$$

Solution:

The additive inverse of a matrix is obtained by changing the sign of each entity. So, according to the definition;

(i) Additive inverse of A = - A = $\begin{bmatrix} -2 & -4 \\ 2 & -1 \end{bmatrix}$

(ii) Additive inverse of B = - B = $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$

(iii) Additive inverse of C = - C = $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$

(iv) Additive inverse of D = - D = $\begin{bmatrix} -1 & 0 \\ 3 & 2 \\ -2 & -1 \end{bmatrix}$

(v) Additive inverse of E = - E = $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(vi) Additive inverse of F = - F = $\begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$

Q3. **If $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$,**
$D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$, then find:

(i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **(ii)** $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

(iii) $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$ **(iv)** $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

(v) **2A** **(vi)** **(-1) B**

(vii) **(-2) C** **(viii)** **3D** **(ix)** **3C**

Solution:

(i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1+1 & 2+1 \\ 2+1 & 1+1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$

So, $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$

(ii) $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+(-2) \\ (-1)+3 \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

So, $B + \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

(iii) $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+(-2) & (-1)+1 & 2+3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 & 5 \end{bmatrix}$

So, $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 5 \end{bmatrix}$

(iv) $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 2+1 & 3+0 \\ -1+2 & 0+0 & 2+1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 3 \end{bmatrix}$

So, $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 3 \end{bmatrix}$

(v) $2A = 2 \times \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 \times (-1) & 2 \times 2 \\ 2 \times 2 & 2 \times 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$

So, $2A = \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$

(vi) $(-1)B = (-1) \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} (-1) \times 1 \\ (-1) \times (-1) \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

So, $(-1)B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(vii) $(-2)C = (-2) \times \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} (-2) \times 1 & (-2) \times -1 & (-2) \times 2 \end{bmatrix}$

$= \begin{bmatrix} -2 & 2 & -4 \end{bmatrix}$

So, $(-2)C = \begin{bmatrix} -2 & 2 & -4 \end{bmatrix}$

(viii) $3D = 3 \times \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3 \times 1 & 3 \times 2 & 3 \times 3 \\ 3 \times (-1) & 3 \times 0 & 3 \times 2 \end{bmatrix}$

$= \begin{bmatrix} 3 & 6 & 9 \\ -3 & 0 & 6 \end{bmatrix}$

So, $3D = \begin{bmatrix} 3 & 6 & 9 \\ -3 & 0 & 6 \end{bmatrix}$

(ix) $3C = 3 \times \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3 \times 1 & 3 \times (-1) & 3 \times 2 \end{bmatrix}$

$= \begin{bmatrix} 3 & -3 & 6 \end{bmatrix}$

So, $3C = \begin{bmatrix} 3 & -3 & 6 \end{bmatrix}$

Q4. Perform the indicated operations and simplify the following.

(i) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 2 & 2 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

(v) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$

(vi) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Solution:

(i) $= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + \begin{bmatrix}0 & 2\\3 & 0\end{bmatrix} + \begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

$= \begin{bmatrix}1+0+1 & 0+2+1\\0+3+1 & 1+0+0\end{bmatrix} = \begin{bmatrix}\mathbf{2} & \mathbf{3}\\\mathbf{4} & \mathbf{1}\end{bmatrix}$

(ii) $= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + \begin{bmatrix}0 & 2\\3 & 0\end{bmatrix} - \begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

$= \begin{bmatrix}1+0-1 & 0+2-1\\0+3-1 & 1+0-0\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{1}\\\mathbf{2} & \mathbf{1}\end{bmatrix}$

(iii) $= \begin{bmatrix}2 & 3 & 1\end{bmatrix} + (\begin{bmatrix}1 & 0 & 2\end{bmatrix} - \begin{bmatrix}2 & 2 & 2\end{bmatrix})$

$= \begin{bmatrix}2+1-2 & 3+0-2 & 1+2-2\end{bmatrix}$

$= \begin{bmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1}\end{bmatrix}$

(iv) $= \begin{bmatrix}1 & 2 & 3\\-1 & -1 & -1\\0 & 1 & 2\end{bmatrix} + \begin{bmatrix}1 & 1 & 1\\2 & 2 & 2\\3 & 3 & 3\end{bmatrix}$

$= \begin{bmatrix}1+1 & 2+1 & 3+1\\-1+2 & -1+2 & -1+2\\0+3 & 1+3 & 2+3\end{bmatrix} = \begin{bmatrix}\mathbf{2} & \mathbf{3} & \mathbf{4}\\\mathbf{1} & \mathbf{1} & \mathbf{1}\\\mathbf{3} & \mathbf{4} & \mathbf{5}\end{bmatrix}$

(v) $= \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & 1 & 2\end{bmatrix} + \begin{bmatrix}1 & 0 & -2\\-2 & -1 & 0\\0 & 2 & -1\end{bmatrix}$

$= \begin{bmatrix}1+1 & 2+0 & 3+(-2)\\2+(-2) & 3+(-1) & 1+0\\3+0 & 1+2 & 2+(-1)\end{bmatrix}$

$= \begin{bmatrix}\mathbf{2} & \mathbf{2} & \mathbf{1}\\\mathbf{0} & \mathbf{2} & \mathbf{1}\\\mathbf{3} & \mathbf{3} & \mathbf{1}\end{bmatrix}$

(vi) $= \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}2 & 1\\0 & 0\end{bmatrix} + \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

$= \begin{bmatrix}1+2+1 & 2+1+1\\0+0+1 & 1+0+1\end{bmatrix} = \begin{bmatrix}4 & 4\\2 & 2\end{bmatrix}$

Q5. For the matrices $A = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\0 & 2 & -1\end{bmatrix}$, $B = \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$

and $C = \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$ verify the following rules.

(i) $A + C = C + A$ **(ii)** $A + B = B + A$

(iii) $B + C = C + B$ **(iv)** $A + (B + A) = 2A + B$

(v) $(C - B) + A + C + (A - B)$

(vi) $2A + B = A + (A + B)$

(vii) $(C - B) - A = (C - A) - B$

(viii) $(A + B) + C = A + (B + C)$

(ix) $A + (B - C) = (A - C) + B$

(x) **$2A + 2B = 2(A + B)$**

(i) $A + C = C + A$

Solution:

$$\text{L.H.S} = A + C$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(-1) & 2+0 & 3+0 \\ 2+0 & 3+(-2) & 1+3 \\ 1+1 & -1+1 & 0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 0 & 2 \end{bmatrix} \quad \text{----------------- (1)}$$

$$\text{R.H.S} = C + A$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 0+2 & 0+3 \\ 0+2 & (-2)+3 & 3+1 \\ 1+1 & 1+-1 & 2+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 0 & 2 \end{bmatrix} \quad \text{----------------- (2)}$$

From "1" and "2", it is proved that: **$A + C = C + A$**

(ii) $A + B = B + A$

Solution:

$$\text{L.H.S} = A + B$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+(-1) & 3+1 \\ 2+2 & 3+(-2) & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} \quad \text{----------------- (1)}$$

$$\text{R.H.S} = B + A$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & (-1)+2 & 1+3 \\ 2+2 & (-2)+3 & 2+1 \\ 3+1 & 1+(-1) & 3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} \quad \text{-------------- (2)}$$

From "1" and "2", it is proved that: **A + B = B + A**

(iii) B + C = C + B

Solution:

L.H.S = B + C

$$= \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(-1) & -1+0 & 1+0 \\ 2+(-2) & -2+(-2) & 2+3 \\ 3+1 & 1+1 & 3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 0 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix} \quad \text{---------- (1)}$$

R.H.S = C + B

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 0+(-1) & 0+1 \\ 0+2 & -2+(-2) & 3+2 \\ 1+3 & 1+1 & 2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix} \quad \text{------------ (2)}$$

From "1" and "2", it is proved that: **B + C = C + B**

(iv) A + (B + A) = 2A + B

Solution:

L.H.S = A + (B + A)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1+1+1 & 2+(-1)+2 & 3+1+3 \\ 2+2+2 & 3+(-2)+3 & 1+2+1 \\ 1+3+1 & -1+1+(-1) & 0+3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix} \quad \text{--------------- (1)}$$

$$\text{R.H.S} = 2A + B$$

$$= 2 \times \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 1 & 2 \times 2 & 2 \times 3 \\ 2 \times 2 & 2 \times 3 & 2 \times 1 \\ 2 \times 1 & 2 \times (-1) & 2 \times 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 4+(-1) & 6+1 \\ 4+2 & 6+(-2) & 2+2 \\ 2+3 & (-2)+1 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix} \quad \text{------------ (2)}$$

From "1" and "2" it is proved that: **A + (B + A) = 2A**

(v) (C – B) + A = (C – A) – B

Solution:

$$\text{L.H.S} = (C - B) + A$$

$$= \left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right) + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & 0-(-1) & 0-1 \\ 0-2 & -2-(-2) & 3-2 \\ 1-3 & 1-1 & 2-3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & -1 \\ -2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2+1 & 1+2 & -1+3 \\ -2+2 & 0+3 & 1+1 \\ -2+1 & 0+(-1) & -1+0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 & 2 \\ 0 & 3 & 2 \\ -1 & -1 & -1 \end{bmatrix} \quad \text{------------ (1)}$$

$$\text{R.H.S} = C + (A - B)$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & & 2 \end{bmatrix} + \begin{bmatrix} 1-1 & 2-(-1) & 3-1 \\ 2-2 & 3-(-2) & 1-2 \\ 1-3 & -1-1 & 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 & 2 \\ 0 & 5 & -1 \\ -2 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0 & 0+3 & 0+2 \\ 0+0 & -2+5 & 3+(-1) \\ 1+(-2) & 1+(-2) & 2+(-3) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 & 2 \\ 0 & 3 & 2 \\ -1 & -1 & -1 \end{bmatrix} \quad \text{------------- (2)}$$

From "1" and "2", it is proved that: **(C – B) + A = C + (A – B)**

(vi) 2A + B = A + (A + B)

Solution:

L.H.S = 2A + B

$$= 2 \times \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1 & 2\times2 & 2\times3 \\ 2\times2 & 2\times3 & 2\times1 \\ 2\times1 & 2\times(-1) & 2\times0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 4+(-1) & 6+1 \\ 4+2 & 6+(-2) & 2+2 \\ 2+3 & (-2)+1 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix} \quad \text{------------ (1)}$$

R.H.S = A + (A + B)

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1+1 & 2+(-1) & 3+1 \\ 2+2 & 3+(-2) & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2+1 & 3+4 \\ 2+4 & 3+1 & 1+3 \\ 1+4 & -1+0 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix} \quad \text{----------- (2)}$$

From "1" and "2", it is proved that: **2A + B = A + (A + B)**

(vii) (C − B) − A = (C − A) − B

Solution:

L.H.S = (C − B) − A

$$= \left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}\right) - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & 0-(-1) & 0-1 \\ 0-2 & -2-(-2) & 3-2 \\ 1-3 & 1-1 & 2-3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & -1 \\ -2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2-1 & 1-2 & -1-3 \\ -2-2 & 0-3 & 1-1 \\ -2-1 & 0-(-1) & -1-0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 & -4 \\ -4 & -3 & 0 \\ -3 & 1 & -1 \end{bmatrix} \quad \text{----------- (1)}$$

R.H.S = (C − A) − B

$$= \left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}\right) - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & 0-2 & 0-3 \\ 0-2 & -2-3 & 3-1 \\ 1-1 & 1-(-1) & 2-0 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 & -3 \\ -2 & -5 & 2 \\ 0 & 2 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2-1 & -2-(-1) & -3-1 \\ -2-2 & -5-(-2) & 2-2 \\ 0-3 & 2-1 & 2-3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 & -4 \\ -4 & -3 & 0 \\ -3 & 1 & -1 \end{bmatrix} \quad \text{----------- (2)}$$

From "1" and "2", it is proved that: **(C − B) − A = (C − A) − B**

(viii) (A + B) + C = A + (B + C)

Solution:

L.H.S = (A + B) + C

$$= \left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$= \begin{bmatrix}1+1 & 2+(-1) & 3+1\\2+2 & 3+(-2) & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$= \begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$= \begin{bmatrix}2+(-1) & 1+0 & 4+0\\4+0 & 1+(-2) & 3+3\\4+1 & 0+1 & 3+2\end{bmatrix}$$

$$= \begin{bmatrix}1 & 1 & 4\\4 & -1 & 6\\5 & 1 & 5\end{bmatrix} \quad \text{----------- (1)}$$

R.H.S $=$ $A + (B + C)$

$$= \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}\right)$$

$$= \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1+(-1) & -1+0 & 1+0\\2+0 & -2+(-2) & 2+3\\3+1 & 1+1 & 3+2\end{bmatrix}$$

$$= \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}0 & -1 & 1\\2 & -4 & 5\\4 & 2 & 5\end{bmatrix}$$

$$= \begin{bmatrix}1+0 & 2+(-1) & 3+1\\2+2 & 3+(-4) & 1+5\\1+4 & -1+2 & 0+5\end{bmatrix}$$

$$= \begin{bmatrix}1 & 1 & 4\\4 & -1 & 6\\5 & 1 & 5\end{bmatrix} \quad \text{----------- (2)}$$

From "1" and "2", it is proved that: **$(A + B) + C = A + (B + C)$**

(ix) $A + (B - C) = (A - C) + B$

Solution:

L.H.S $=$ $A + (B - C)$

$$= \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}-\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}\right)$$

$$= \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1-(-1) & -1-0 & 1-0\\2-0 & -2-(-2) & 2-3\\3-1 & 1-1 & 3-2\end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 2+(-1) & 3+1 \\ 2+2 & 3+0 & 1+(-1) \\ 1+2 & -1+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix} \text{ ---------- (1)}$$

R.H.S $= (A - C) + B$

$$= \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}\right) + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-(-1) & 2-0 & 3-0 \\ 2-0 & 3-(-2) & 1-3 \\ 1-1 & -1-1 & 0-2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 3 \\ 2 & 5 & -2 \\ 0 & -2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 2+(-1) & 3+1 \\ 2+2 & 5+(-2) & -2+2 \\ 0+3 & -2+1 & -2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix} \text{ ---------- (2)}$$

From "1" and "2", it is proved that: **$A + (B - C) = (A - C) + B$**

(x) $2A + 2B = 2(A + B)$

Solution:

L.H.S $= 2A + 2B$

$$= \left(2 \times \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}\right) + \left(2 \times \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2\times1 & 2\times2 & 2\times3 \\ 2\times2 & 2\times3 & 2\times1 \\ 2\times1 & 2\times(-1) & 2\times0 \end{bmatrix} + \begin{bmatrix} 2\times1 & 2\times(-1) & 2\times1 \\ 2\times2 & 2\times(-2) & 2\times2 \\ 2\times3 & 2\times1 & 2\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 & 2 \\ 4 & -4 & 4 \\ 6 & 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 4+(-2) & 6+2 \\ 4+4 & 6+(-4) & 2+4 \\ 2+6 & -2+2 & 0+6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 8 \\ 8 & 2 & 6 \\ 8 & 0 & 6 \end{bmatrix} \quad \text{------------- (1)}$$

R.H.S $= 2(A + B)$

$$= 2 \times \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right)$$

$$= 2 \times \begin{bmatrix} 1+1 & 2+(-1) & 3+1 \\ 2+2 & 3+(-2) & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix}$$

$$= 2 \times \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times 2 & 2\times 1 & 2\times 4 \\ 2\times 4 & 2\times 1 & 2\times 3 \\ 2\times 4 & 2\times 0 & 2\times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 8 \\ 8 & 2 & 6 \\ 8 & 0 & 6 \end{bmatrix} \quad \text{----------- (2)}$$

From "1" and "2", it is proved that: **$2A + 2B = 2(A + B)$**

Q6. If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$, find (i) $3A - 2B$ (ii) $2A^t - 3B^t$.

Solution:

(i) $3A - 2B$

$$= 3 \times \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} - 2 \times \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times 1 & 3\times(-2) \\ 3\times 3 & 3\times 4 \end{bmatrix} - \begin{bmatrix} 2\times 0 & 2\times 7 \\ 2\times(-3) & 2\times 8 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -6 \\ 9 & 12 \end{bmatrix} - \begin{bmatrix} 0 & 14 \\ -6 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 3-0 & (-6)-(14) \\ 9-(-6) & 12-16 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -20 \\ 15 & -4 \end{bmatrix}$$

So, $3A - 2B = \begin{bmatrix} 3 & -20 \\ 15 & -4 \end{bmatrix}$

(ii) $2A^t - 3B^t$

Solution: $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$

$A^t = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$, $B^t = \begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$

$2A^t = 2 \times \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$, $3B^t = 3 \times \begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$

$$2A^t = \begin{bmatrix} 2\times 1 & 2\times 3 \\ 2\times(-2) & 2\times 4 \end{bmatrix}, \qquad 3B^t = \begin{bmatrix} 3\times 0 & 3\times(-3) \\ 3\times 7 & 3\times 8 \end{bmatrix}$$

$$2A^t = \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix}, \qquad 3B^t = \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix}$$

$$2A^t - 3B^t = \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 6-(-9) \\ -4-21 & 8-24 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 15 \\ -25 & -16 \end{bmatrix}$$

So, $\mathbf{2A^t - 3B^t} = \begin{bmatrix} 2 & 15 \\ -25 & -16 \end{bmatrix}$

Q7. If $2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$, then find a and b.

Solution:

Given $2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$

$$\text{L.H.S} = 2\times\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\times\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times 2 & 2\times(4) \\ 2\times(-3) & 2\times a \end{bmatrix} + \begin{bmatrix} 3\times 1 & 3\times b \\ 3\times 8 & 3\times(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 \\ -6 & 2a \end{bmatrix} + \begin{bmatrix} 3 & 3b \\ 24 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & -8+3b \\ (-6)+24 & 2a-12 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 8+3b \\ 18 & 2a-12 \end{bmatrix}$$

By equating it with R.H.S, we have:

$$\begin{bmatrix} 7 & 8+3b \\ 18 & 2a-12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

By comparing corresponding elements

$8 + 3b = 10$

$3b = 10 - 8$

$3b = 2$

$b = \frac{2}{3}$ -------- (eq-1)

$2a - 12 = 1$

$2a = 1 + 12$

$2a = 13$

$a = \frac{13}{2}$ ---------- (eq-2)

From equations "1" and "2", we get

$\mathbf{a = \frac{13}{2}}$ **and** $\mathbf{b = \frac{2}{3}}$

Q8. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$, then verify that:

(i) $(A + B)^t = A^t + B^t$ **(ii) $(A - B)^t = A^t - B^t$**

(iii) $A + A^t$ is symmetric

(iv) $A - A^t$ is skew symmetric

(v) $B + B^t$ is symmetric

(vi) $B - B^t$ is skew symmetric

(i) $(A + B)^t = A^t + B^t$

Solution:

$$\text{L.H.S} = (A + B)^t$$

$$= \left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}\right)^t$$

$$= \left(\begin{bmatrix} 1+1 & 2+1 \\ 0+2 & 1+0 \end{bmatrix}\right)^t$$

$$= \left(\begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}\right)^t$$

$$= \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} \quad \text{----------- (i)}$$

$$\text{R.H.S} = A^t + B^t$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 0+2 \\ 2+1 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} \quad \text{-------------- (ii)}$$

From (i) and (ii), it is proved that: $(A + B)^t = A^t + B^t$

(ii) $(A - B)^t = A^t - B^t$

Solution:

$$\text{L.H.S} = (A - B)^t$$

$$= \left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}\right)^t$$

$$= \begin{bmatrix} 1-1 & 2-1 \\ 0-2 & 1-0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix} \quad \text{----------- (i)}$$

$$\text{R.H.S} = A^t - B^t$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} - \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$= \begin{bmatrix}1-1 & 0-2\\2-1 & 2-0\end{bmatrix}$$

$$= \begin{bmatrix}0 & -2\\1 & 2\end{bmatrix} \qquad \text{------------- (ii)}$$

From (i) and (ii), it is proved that: **$(A - B)^t = A^t - B^t$**

(iii) To prove $A + A^t$ is symmetric

Solution:

$$A + A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t$$

$$= \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1+1 & 2+0\\0+2 & 1+1\end{bmatrix}$$

$$= \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} \qquad \text{------------- (1)}$$

Now we will take transpose of $A + A^t$

$$(A + A^t)^t = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}^t$$

$$= \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} \qquad \text{------------- (2)}$$

From "1" and "2", it is proved that: $A + A^t = (A + A^t)^t$

So, it is Symmetric.

(iv) To prove $A - A^t$ is skew symmetric

Solution:

$$A - A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t$$

$$= \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1-1 & 2-0\\0-2 & 1-1\end{bmatrix}$$

$$= \begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix} \qquad \text{----------- (i)}$$

Now take the transpose of (i), we have:

$$(A - A^t)^t = \begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}^t$$

$$= \begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}$$

$$= (-1)\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}$$

$$= -\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix} \qquad \text{--------- (ii)}$$

$$= -(A - A^t)$$

From (i) and (ii), it is obvious that:

$A - A^t$ is skew symmetric

(v) To prove $B + B^t$

Solution:

$$B + B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 1+2 \\ 2+1 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix} \text{ -------------- (i)}$$

Taking transpose of (i) we have:

$$(B + B^t)^t = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix} \text{ -------------- (ii)}$$

From (i) and (ii), it is obvious that: **$B + B^t$ is symmetric**

(vi) To prove $B - B^t$ is skew symmetric

Solution:

$$B - B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 1-2 \\ 2-1 & 0-0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \text{ ------------ (i)}$$

Now taking transpose of (i), we have:

$$(B - B^t)^t = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= (-1)\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= -1\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \text{ ------------- (ii)}$$

$$= -(B - B^t)$$

From (i) and (ii), it is obvious that:

$B - B^t$ is skew symmetric

EXERCISE 1.4

Q1. Which of the following product of matrices is conformable for multiplication

(i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$ **(iv)** $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Solution:

Two matrices are conformable for multiplication if the numbers of columns of first matrix are equal to number of rows of second matrix.

So, according to the definition:

(i) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows).

(ii) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows).

(iii) is not conformable for multiplication (because the first matrix has just one column and second matrix has two rows).

(iv) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows).

(v) is conformable for multiplication (because the first matrix has three columns and second matrix has same number of rows).

Q2. If $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$, find (i) AB (ii) BA (if possible)

Solution:

(i) AB

$= \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$= \begin{bmatrix} 3 \times 6 + 0 \times 5 \\ (-1) \times 6 + 2 \times 5 \end{bmatrix}$

EXERCISE 1.4

Q1. Which of the following product of matrices is conformable for multiplication

(i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$ **(iv)** $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Solution:

Two matrices are conformable for multiplication if the numbers of columns of first matrix are equal to number of rows of second matrix.

So, according to the definition:

(i) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows).

(ii) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows).

(iii) is not conformable for multiplication (because the first matrix has just one column and second matrix has two rows).

(iv) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows).

(v) is conformable for multiplication (because the first matrix has three columns and second matrix has same number of rows).

Q2. If $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$, find (i) AB (ii) BA (if possible)

Solution:

(i) AB

$= \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$= \begin{bmatrix} 3 \times 6 + 0 \times 5 \\ (-1) \times 6 + 2 \times 5 \end{bmatrix}$

$$= \begin{bmatrix} 18+0 \\ -6+10 \end{bmatrix} = \begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

So, **AB** $= \begin{bmatrix} 18 \\ 4 \end{bmatrix}$

(ii) **BA**

BA is not possible (because number of columns of B is not equal to number of rows of A)

Q3. Find the following products

(i) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix}$

(iii) $\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ **(iv)** $\begin{bmatrix} 6 & -0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

(v) $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

(i) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Solution:

$$= [1 \times 4 + 2 \times 0] = [4+0]$$
$$= [4]$$

So, $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix} = [4]$

(ii) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix}$

Solution:

$$= [1 \times 5 + 2 \times (-4)] = [5-8]$$
$$= [-3]$$

So, $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix} = [-3]$

(iii) $\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Solution:

$$= [(-3) \times 4 + 0 \times 0] = [-12]$$

So, $\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix} = [-12]$

(iv) $\begin{bmatrix} 6 & -0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Solution:

$$= [6 \times 4 + 0 \times 0] = [24+0]$$
$$= [24]$$

So, $\begin{bmatrix} 6 & -0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix} = [24]$

(v) $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$

Solution:

$$= \begin{bmatrix}1\times4+2\times0 & 1\times5+2\times(-4)\\-3\times4+0\times0 & -3\times5+0\times(-4)\\6\times4+(-1)\times0 & 6\times5+(-1)\times(-4)\end{bmatrix}$$

$$= \begin{bmatrix}4+0 & 5+(-8)\\-12+0 & -15+0\\24+0 & 30+4\end{bmatrix}$$

$$= \begin{bmatrix}4 & -3\\-12 & -15\\24 & 34\end{bmatrix}$$

So, $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix} = \begin{bmatrix}4 & -3\\-12 & -15\\24 & 34\end{bmatrix}$

Q4. Multiply the following matrices.

(a) $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$ **(b)** $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2\\3 & 4\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$ **(d)** $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -5/2\\-4 & 4\end{bmatrix}$

(e) $\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

(a) $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$

Solution:

$$= \begin{bmatrix}2\times2+3\times3 & 2\times(-1)+3\times0\\1\times2+1\times3 & 1\times(-1)+1\times0\\0\times2+(-2)\times3 & 0\times(-1)+(-2)\times0\end{bmatrix}$$

$$= \begin{bmatrix}4+9 & -2+0\\2+3 & -1+0\\0-6 & 0+0\end{bmatrix}$$

$$= \begin{bmatrix}13 & -2\\5 & -1\\-6 & 0\end{bmatrix}$$

So, $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix} = \begin{bmatrix}13 & -2\\5 & -1\\-6 & 0\end{bmatrix}$

(b) $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

Solution:

$$= \begin{bmatrix}1\times1+2\times3+3\times(-1) & 1\times2+2\times4+3\times1\\4\times1+5\times3+6\times(-1) & 4\times2+5\times4+6\times1\end{bmatrix}$$

$$= \begin{bmatrix}1+6-3 & 2+8+3\\4+15-6 & 8+20+6\end{bmatrix}$$

$$= \begin{bmatrix}4 & 13\\13 & 34\end{bmatrix}$$

So, $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix} = \begin{bmatrix}4 & 13\\13 & 34\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$

Solution:

$$= \begin{bmatrix}1\times1+2\times4 & 1\times2+2\times5 & 1\times3+2\times6\\3\times1+4\times4 & 3\times2+4\times5 & 3\times3+4\times6\\-1\times1+1\times4 & -1\times2+1\times5 & -1\times3+1\times6\end{bmatrix}$$

$$= \begin{bmatrix}1+8 & 2+10 & 3+12\\3+16 & 6+20 & 9+24\\-1+4 & -2+5 & -3+6\end{bmatrix}$$

$$= \begin{bmatrix}9 & 12 & 15\\19 & 26 & 33\\3 & 3 & 3\end{bmatrix}$$

So, $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} = \begin{bmatrix}9 & 12 & 15\\19 & 26 & 33\\3 & 3 & 3\end{bmatrix}$

(d) $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -5/2\\-4 & 4\end{bmatrix}$

Solution:

$$= \begin{bmatrix}8\times2+5\times(-4) & 8\times\left(-\frac{5}{2}\right)+5\times4\\6\times2+4\times(-4) & 6\times\left(-\frac{5}{2}\right)+4\times4\end{bmatrix}$$

$$= \begin{bmatrix}16-20 & -20+20\\12-16 & -15+16\end{bmatrix}$$

$$= \begin{bmatrix}-4 & 0\\-4 & 1\end{bmatrix}$$

So, $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -5/2\\-4 & 4\end{bmatrix} = \begin{bmatrix}-4 & 0\\-4 & 1\end{bmatrix}$

(e) $\begin{bmatrix}-1 & 2\\ 1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

Solution:

$$= \begin{bmatrix}-1\times 0+2\times 0 & -1\times 0+2\times 0\\ 1\times 0+3\times 0 & 1\times 0+3\times 0\end{bmatrix}$$

$$= \begin{bmatrix}0+0 & 0+0\\ 0+0 & 0+0\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

So, $\begin{bmatrix}-1 & 2\\ 1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

Q5. Let $A = \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$, $B = \begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$ **and** $C = \begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$.

Verify whether

(i) AB = BA **(ii) A (BC) = (AB) C**

(iii) A (B + C) = AB + AC

(iv) A (B − C) = AB − AC

(i) AB=BA

Solution:

L.H.S = AB

$$= \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$= \begin{bmatrix}-1\times 1 & 3\times 2\\ 2\times(-3) & 0\times(-5)\end{bmatrix}$$

$$= \begin{bmatrix}-1 & 6\\ -6 & 0\end{bmatrix}$$

R.H.S = BA

$$= \begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

$$= \begin{bmatrix}1\times(-1) & 2\times 3\\ -3\times 2 & -5\times 0\end{bmatrix}$$

$$= \begin{bmatrix}-1 & 6\\ -6 & 0\end{bmatrix}$$

Therefore L.H.S = R.H.S

AB = BA

(ii) A (BC) = (AB) C

Solution:

L.H.S = A(BC)

$$= \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$= \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1\times 2+2\times 1 & 1\times 1+2\times 3\\ -3\times 2+(-5)\times 1 & -3\times 1+(-5)\times 3\end{bmatrix}\right)$$

$$= \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2+2 & 1+6\\ -6-5 & -3-15\end{bmatrix}$$

$$= \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}4 & 7\\ -11 & -18\end{bmatrix}$$

$$= \begin{bmatrix} -1\times 4+3\times(-11) & -1\times 7+3\times(-18) \\ 2\times 4+0\times(-11) & 2\times 7+0\times(-18) \end{bmatrix}$$

$$= \begin{bmatrix} -4-33 & -7-54 \\ 8+0 & 14+0 \end{bmatrix}$$

$$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix} \text{----------- (i)}$$

R.H.S $=$ (AB) C

$$= \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\right) \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1\times 1+3\times(-3) & -1\times 2+3\times(-5) \\ 2\times 1+0\times(-3) & 2\times 2+0\times(-5) \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -10\times 2+(-17)\times 1 & (-10)\times 1+(-17)\times 3 \\ 2\times 2+4\times 1 & 2\times 1+4\times 3 \end{bmatrix}$$

$$= \begin{bmatrix} -20-17 & -10-51 \\ 4+4 & 2+12 \end{bmatrix}$$

$$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix} \text{------------ (ii)}$$

From (i) and (ii), it is obvious that: L.H.S = R.H.S

A (BC) = (AB) C

(iii) A (B + C) = AB + AC

Solution:

L.H.S $=$ A (B + C)

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1+2 & 2+1 \\ -3+1 & -5+3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -1\times 3+3\times(-2) & -1\times 3+3\times(-2) \\ 2\times 3+0\times(-2) & 2\times 3+0\times(-2) \end{bmatrix}$$

$$= \begin{bmatrix} -3-6 & -3-6 \\ 6+0 & 6+0 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix} \text{---------- (i)}$$

R.H.S $=$ AB + AC

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1\times 1+3\times(-3) & -1\times 2+3\times(-5) \\ 2\times 1+0\times(-3) & 2\times 2+0\times(-5) \end{bmatrix}$$

$$+\begin{bmatrix}-1\times2+3\times1 & -1\times1+3\times3\\ 2\times2+0\times1 & 2\times1+0\times3\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}+\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10+1 & -17+8\\ 2+4 & 4+2\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix} \text{ ---------- (ii)}$$

From (i) and (ii), it is proved that: L.H.S = R.H.S

A (B + C) = AB + AC

(iv) A (B – C) = AB – AC

Solution:

L.H.S = A (B – C)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1-2 & 2-1\\ -3-1 & -5-3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}-1 & 1\\ -4 & -8\end{bmatrix}$$

$$=\begin{bmatrix}-1\times(-1)+3\times(-4) & -1\times1+3\times(-8)\\ 2\times(-1)+0\times(-4) & 2\times1+0\times(-8)\end{bmatrix}$$

$$=\begin{bmatrix}1-12 & -1-24\\ -2+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}11 & -25\\ -2 & 2\end{bmatrix} \text{ --------- (i)}$$

R.H.S = AB – AC

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1\times1+3\times(-3) & -1\times2+3\times(-5)\\ 2\times1+0\times(-3) & 2\times2+0\times(-5)\end{bmatrix}$$

$$-\begin{bmatrix}-1\times2+3\times1 & -1\times1+3\times3\\ 2\times2+0\times1 & 2\times1+0\times3\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}-\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}-\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10-1 & -17-8\\ 2-4 & 4-2\end{bmatrix}$$

$$=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix} \text{ ----------- (ii)}$$

From (i) and (ii), hence proved: L.H.S = R.H.S

A (B – C) = AB – AC

Q6. For the matrices

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}, C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$$

Verify that (i) $(AB)^t = B^t A^t$ (ii) $(BC)^t = C^t B^t$

(i) $(AB)^t = B^t A^t$

Solution:

$$\text{L.H.S} = (AB)^t$$

$$= \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\right)^t$$

$$= \left(\begin{bmatrix} -1 \times 1 + 3 \times (-3) & -1 \times 2 + 3 \times (-5) \\ 2 \times 1 + 0 \times (-3) & 2 \times 2 + 0 \times (-5) \end{bmatrix}\right)^t$$

$$= \left(\begin{bmatrix} -1 - 9 & -2 - 15 \\ 2 + 0 & 4 + 0 \end{bmatrix}\right)^t$$

$$= \left(\begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}\right)^t$$

$$= \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix} \quad \text{------------- (i)}$$

$$\text{R.H.S} = B^t A^t$$

$$= \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}^t \times \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times (-1) + (-3) \times 3 & 1 \times 2 + (-3) \times 0 \\ 2 \times (-1) + (-5) \times 3 & 2 \times 2 + (-5) \times 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 - 9 & 2 + 0 \\ -2 - 15 & 4 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix} \quad \text{------------- (ii)}$$

From (i) and (ii), it is proved that: L.H.S = R.H.S

$$(AB)^t = B^t A^t$$

(ii) $(BC)^t = C^t B^t$

Solution:

$$\text{L.H.S} = (BC)^t$$

$$= \left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \times \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}\right)^t$$

$$= \begin{bmatrix} 1 \times (-2) + 2 \times 3 & 1 \times 6 + 2 \times (-9) \\ -3 \times (-2) + (-5) \times 3 & -3 \times 6 + (-5) \times (-9) \end{bmatrix}^t$$

$$= \begin{bmatrix} -2 + 6 & 6 - 18 \\ 6 - 15 & -18 + 45 \end{bmatrix}^t$$

$$= \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix}^t$$

$$= \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix} \text{ ------------ (i)}$$

$$\text{R.H.S} = C^tB^t$$

$$= \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}^t \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}^t$$

$$= \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -2\times 1 + 3\times 2 & -2\times(-3) + 3\times(-5) \\ 6\times 1 + (-9)\times 2 & 6\times(-3) + (-9)\times(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -2+6 & 6-15 \\ 6-18 & -18+45 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix} \text{ ------------ (ii)}$$

From (i) and (ii), hence proved: L.H.S = R.H.S

$$(BC)^t = C^tB^t$$

EXERCISE 1.5

Q1. Find the determinant of the following matrices.

(i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$ **(ii)** $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

(iii) $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$ **(iv)** $D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Solution:

(i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

Determinant of matrix A is calculated as:

$$|A| = \det A = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix} = (-1)\times 0 - 2\times 1$$

$$\mathbf{|A| = 0 - 2 = -2}$$

(ii) $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

Determinant of matrix B is calculated as:

$$|B| = \det B = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} = 1\times(-2) - 2\times 3$$

$$\mathbf{|B| = -2 - 6 = -8}$$

(iii) $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

Determinant of matrix C is calculated as:

$$|C| = \det C = \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix} = 3\times 2 - 3\times 2$$

$$\mathbf{|C| = 6 - 6 = 0}$$

$$= \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix} \text{ ------------ (i)}$$

R.H.S $= C^tB^t$

$$= \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}^t \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}^t$$

$$= \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \times 1 + 3 \times 2 & -2 \times (-3) + 3 \times (-5) \\ 6 \times 1 + (-9) \times 2 & 6 \times (-3) + (-9) \times (-5) \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 6 & 6 - 15 \\ 6 - 18 & -18 + 45 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix} \text{ ------------ (ii)}$$

From (i) and (ii), hence proved: L.H.S = R.H.S

$(BC)^t = C^tB^t$

EXERCISE 1.5

Q1. Find the determinant of the following matrices.

(i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$ **(ii)** $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

(iii) $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$ **(iv)** $D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Solution:

(i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

Determinant of matrix A is calculated as:

$$|A| = \det A = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix} = (-1) \times 0 - 2 \times 1$$

$$\mathbf{|A| = 0 - 2 = -2}$$

(ii) $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

Determinant of matrix B is calculated as:

$$|B| = \det B = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} = 1 \times (-2) - 2 \times 3$$

$$\mathbf{|B| = -2 - 6 = -8}$$

(iii) $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

Determinant of matrix C is calculated as:

$$|C| = \det C = \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix} = 3 \times 2 - 3 \times 2$$

$$\mathbf{|C| = 6 - 6 = 0}$$

(iv) $\mathbf{D} = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Determinant of matrix D is calculated as:

$|D| = \det D = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 3 \times 4 - 2 \times 1$

$|A| = 12 - 2 = 10$

Q2. Find which of the following matrices are singular or non-singular?

(i) $\mathbf{A} = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ **(ii)** $\mathbf{B} = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

(iii) $\mathbf{C} = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$ **(iv)** $\mathbf{D} = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

(i) $\mathbf{A} = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

Solution:

A matrix is said to be singular if its determinant is equal to zero. i.e. $|A| = 0$.

Determinant of matrix A is calculated as:

$|A| = \det A = \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix} = 3 \times 4 - 2 \times 6$

$|A| = 12 - 12 = 0$

As, determinant of A is equal to zero so, A is a singular matrix.

(ii) $\mathbf{B} = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

Solution:

Determinant of matrix B is calculated as:

$|B| = \det B = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} = 4 \times 2 - 3 \times 1$

$|A| = 8 - 3 = 5 \neq 0$

As, determinant of B is not equal to zero so, B is a non singular matrix.

(iii) $\mathbf{C} = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$

Solution:

Determinant of matrix C is calculated as:

$|C| = \det C = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix} = 7 \times 5 - 3 \times (-9)$

$|C| = 35 + 27 = 62 \neq 0$

As, determinant of C is equal to zero so, C is a non-singular matrix.

(iv) $D = \begin{bmatrix} 5 & 10 \\ -2 & 4 \end{bmatrix}$

Determinant of matrix D is calculated as:

$|D| = \det D = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix} = 5 \times 4 - (-2) \times (-10)$

$|D| = 20 - 20 = 0$

As, determinant of D is equal to zero so, D is a singular matrix.

Q3. Find the multiplicative inverse (if it exists) of each.

(i) $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$ **(ii)** $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

(iii) $C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$ **(iv)** $D = \begin{bmatrix} 1/2 & 3/4 \\ 1 & 2 \end{bmatrix}$

(i) $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

Solution:

The multiplicative inverse of matrix A is calculated as:

$A^{-1} = \frac{Adj\ A}{|A|}$

$\text{Adj } A = \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$

$|A| = (-1) \times 0 - 2 \times (3) = 0 - 6 = -6 \neq 0$

Since it is a non-singular matrix therefore solution is possible.

$$A^{-1} = \frac{\begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}}{-6} = \begin{bmatrix} \frac{0}{-6} & \frac{-3}{-6} \\ \frac{-2}{-6} & \frac{-1}{-6} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

(ii) $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

Solution:

The multiplicative inverse of matrix B is calculated as:

$$B^{-1} = \frac{Adj\,B}{|B|}$$

$$\text{Adj B} = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix}$$

$$|B| = 1 \times (-5) - (-3) \times (2) = -5 + 6 = 1 \neq 0$$

Since it is a non-singular matrix therefore solution is possible

$$B^{-1} = \frac{\begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}}{1} = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

$$\mathbf{B^{-1}} = \begin{bmatrix} \mathbf{-5} & \mathbf{-2} \\ \mathbf{3} & \mathbf{1} \end{bmatrix}$$

(iii) $\mathbf{C} = \begin{bmatrix} \mathbf{-2} & \mathbf{6} \\ \mathbf{3} & \mathbf{-9} \end{bmatrix}$

Solution:

The multiplicative inverse of matrix C is calculated as:

$$C^{-1} = \frac{Adj\,C}{|C|}$$

$$\text{Adj C} = \begin{bmatrix} -9 & -6 \\ -3 & -2 \end{bmatrix}$$

$$|C| = \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix}$$

$$|C| = (-9) \times (-2) - (-3) \times (-6) = 18 - 18 = 0$$

Since it is a singular matrix therefore solution is not possible

$$C^{-1} = \frac{\begin{bmatrix} -9 & -6 \\ -3 & -2 \end{bmatrix}}{0} = \infty$$

C^{-1} does not exist.

(iv) $\mathbf{D} = \begin{bmatrix} \mathbf{1/2} & \mathbf{3/4} \\ \mathbf{1} & \mathbf{2} \end{bmatrix}$

Solution:

The multiplicative inverse of matrix D is calculated as:

$$D^{-1} = \frac{Adj\,D}{|D|}$$

$$\text{Adj D} = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$|D| = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix}$$

$$|D| = \frac{1}{2} \times 2 - 1 \times \frac{3}{4} = 1 - \frac{3}{4}$$

$$= \frac{4-3}{4} = \frac{1}{4} \neq 0$$

Since it is a non-singular matrix therefore solution is possible

$$D^{-1} = \frac{\begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}}{\frac{1}{4}} = \begin{bmatrix} 2 \times 4 & \frac{-3}{4} \times 4 \\ -1 \times 4 & \frac{1}{2} \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

$$D^{-1} = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

Q4. **If $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$, then**

(i) $A(Adj\ A) = (Adj\ A)\ A = (\det A)I$

(ii) $BB^{-1} = I = B^{-1}B$

Solution:

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

(i) $A(Adj\ A) = (Adj\ A)\ A = (\det A)I$

$$\text{Adj } A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix} = 1 \times 6 - 4 \times 2 = 6 - 8 = -2$$

Now, $A\ (Adj\ A)$

$$= \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \times \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 6 + 2 \times (-4) & 1 \times (-2) + 2 \times 1 \\ 4 \times 6 + 6 \times (-4) & 4 \times (-2) + 6 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 - 8 & -2 + 2 \\ 24 - 24 & -8 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \quad \text{---------- (i)}$$

$(Adj\ A)\ A$

$$= \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \times 1 + (-2) \times 4 & 6 \times 2 + (-2) \times 6 \\ -4 \times 1 + 1 \times 4 & -4 \times 2 + 1 \times 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 - 8 & 12 - 12 \\ -4 + 4 & -8 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \quad \text{---------- (ii)}$$

$(\det A)\ I$

$$= \quad (-2)\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$= \quad \begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix} \qquad \text{---------- (iii)}$$

From (i), (ii) and (iii), it is clear that:

$A(Adj\ A) = (Adj\ A)\,A = (det\ A)I$ Hence proved:

(ii) $BB^{-1} = I = B^{-1}B$

Solution:

As, $B^{-1} = \dfrac{Adj\ B}{\det B}$

$$\text{Adj B} = \begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}$$

$$\det B = \begin{vmatrix}3 & -1\\2 & -2\end{vmatrix} = 3\times(-2) - 2\times(-1)$$

$$= -6 + 2 = -4 \neq 0$$

$$B^{-1} = \frac{\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}}{-4} = \begin{bmatrix}-2/-4 & 1/-4\\-2/-4 & 3/-4\end{bmatrix}$$

$$= \begin{bmatrix}1/2 & -1/4\\1/2 & -3/4\end{bmatrix}$$

Now, BB^{-1}

$$= \begin{bmatrix}3 & -1\\2 & -2\end{bmatrix}\times\begin{bmatrix}1/2 & -1/4\\1/2 & -3/4\end{bmatrix}$$

$$= \begin{bmatrix}3\times\left(\frac{1}{2}\right)+(-1)\times\left(\frac{1}{2}\right) & 3\times\left(-\frac{1}{4}\right)+(-1)\times\left(-\frac{3}{4}\right)\\ 2\times\left(\frac{1}{2}\right)+(-2)\times\left(\frac{1}{2}\right) & 2\times\left(-\frac{1}{4}\right)+(-2)\times\left(-\frac{3}{4}\right)\end{bmatrix}$$

$$= \begin{bmatrix}\frac{3}{2}-\left(\frac{1}{2}\right) & -\frac{3}{4}+\frac{3}{4}\\ 1-1 & -\frac{1}{2}+\frac{3}{2}\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I \qquad \text{---------- (i)}$$

Now $B^{-1}B$

$$= \begin{bmatrix}\frac{1}{2} & -\frac{1}{4}\\ \frac{1}{2} & -\frac{3}{4}\end{bmatrix}\times\begin{bmatrix}3 & -1\\2 & -2\end{bmatrix}$$

$$= \begin{bmatrix}\frac{1}{2}\times 3+\left(-\frac{1}{4}\right)\times 2 & \frac{1}{2}\times(-1)+\left(-\frac{1}{4}\right)\times(-2)\\ \frac{1}{2}\times 3+\left(-\frac{3}{4}\right)\times 2 & \frac{1}{2}\times(-1)+\left(-\frac{3}{4}\right)\times(-2)\end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{2} - \frac{1}{2} & -\frac{1}{2} + \frac{1}{2} \\ \frac{3}{2} - \frac{3}{2} & -\frac{1}{2} + \frac{3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3-1}{2} & \frac{-1+1}{2} \\ \frac{3-3}{2} & \frac{-1+3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad = \quad I \quad \text{-------------} \text{ (ii)}$$

From (i) and (ii), it is clear that:

$BB^{-1} = I = B^{-1}B$ Hence proved

Q5. Determine whether the given matrices are multiplicative inverses of each other.

(i) $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ **and** $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ **and** $\begin{bmatrix} -3 & -3 \\ 2 & -1 \end{bmatrix}$

Solution:

(i) Let $A = \begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$, and $B = \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 7 + 5 \times (-4) & 3 \times (-5) + 5 \times 3 \\ 4 \times 7 + 7 \times (-4) & 4 \times (-5) + 7 \times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 21 - 20 & -15 + 15 \\ 28 - 28 & -20 + 21 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \text{ (identity matrix)}$$

Hence the given matrices are multiplicative inverses of each other.

(ii) Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, and $B = \begin{bmatrix} -3 & 3 \\ 2 & -1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times (-3) + 2 \times 2 & 1 \times 2 + 2 \times (-1) \\ 2 \times (-3) + 3 \times 2 & 2 \times 2 + 3 \times (-1) \end{bmatrix}$$

$$= \begin{bmatrix} -3 + 4 & 2 - 2 \\ -6 + 6 & 4 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \text{ (identity matrix)}$$

Hence the given matrices are multiplicative inverses of each other.

Q6. **If** $A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$, $D = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$, **then verify that**

(i) $(AB)^{-1} = B^{-1}A^{-1}$ **(ii)** $(DA)^{-1} = A^{-1}D^{-1}$

Solution:

(i) $(AB)^{-1} = B^{-1}A^{-1}$

As, $B^{-1} = \dfrac{Adj\,B}{\det B}$

$$Adj\,B = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

$$\det B = \begin{vmatrix} -4 & -2 \\ 1 & -1 \end{vmatrix}$$

$$= -4 \times (-1) - 1 \times (-2) = 4 + 2 = 6 \neq 0$$

$$B^{-1} = \frac{\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}}{6} = \begin{bmatrix} \frac{-1}{6} & \frac{2}{6} \\ \frac{-1}{6} & \frac{-4}{6} \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} \frac{-1}{6} & \frac{1}{3} \\ \frac{-1}{6} & \frac{-2}{3} \end{bmatrix} \text{ ---------- (a)}$$

Similarly, $A^{-1} = \dfrac{Adj\,A}{\det A}$

$$Adj\,A = \begin{bmatrix} 2 & -0 \\ 1 & 4 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix} = 4 \times 2 - (-1) \times 0$$

$$= 8 + 0 = 8 \neq 0$$

$$A^{-1} = \frac{\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}}{8} = \begin{bmatrix} \frac{2}{8} & \frac{0}{8} \\ \frac{1}{8} & \frac{4}{8} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix} \text{ ------------ (b)}$$

Now by solving L.H.S

$$= (AB)$$

$$= \left(\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 4 \times (-4) + 0 \times 1 & 4 \times (-2) + 0 \times (-1) \\ -1 \times (-4) + 2 \times 1 & (-1) \times (-2) + 2 \times (-1) \end{bmatrix}$$

$$= \begin{bmatrix} -16 + 0 & -8 + 0 \\ 4 + 2 & 2 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & -8 \\ 6 & 0 \end{bmatrix}$$

As, $(AB)^{-1} = \dfrac{Adj\ AB}{\det AB}$

$$\text{Adj } AB = \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$\det AB = \begin{vmatrix} -16 & -8 \\ 6 & 0 \end{vmatrix} = -16 \times 0 - 6 \times (-8)$$

$$= 0 + 48 = 48 \neq 0$$

So, $$(AB)^{-1} = \frac{\begin{bmatrix} 0 & -8 \\ 6 & -16 \end{bmatrix}}{48} = \begin{bmatrix} \frac{0}{48} & \frac{-8}{48} \\ \frac{6}{48} & \frac{-16}{48} \end{bmatrix}$$

$$(AB)^{-1} = \begin{bmatrix} 0 & \frac{-1}{6} \\ \frac{1}{8} & \frac{-1}{3} \end{bmatrix} \text{ ----------- (i)}$$

Now by solving R.H.S

$$= B^{-1}A^{-1}$$

$$= \begin{bmatrix} \frac{-1}{6} & \frac{1}{3} \\ \frac{-1}{6} & \frac{-2}{3} \end{bmatrix} \times \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{6} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{8} & \frac{-1}{6} \times 0 + \frac{1}{3} \times \frac{1}{2} \\ \frac{-1}{6} \times \frac{1}{4} + \frac{-2}{3} \times \frac{1}{8} & \frac{-1}{6} \times 0 + \frac{-2}{3} \times \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{24} + \frac{1}{24} & 0 + \frac{1}{6} \\ \frac{-1}{24} - \frac{1}{12} & 0 - \frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & \frac{-1}{3} \end{bmatrix} \text{ -------------- (ii)}$$

From (i) and (ii), it is clear that:

$(AB)^{-1} = B^{-1}A^{-1}$ Hence proved

(ii) $(DA)^{-1} = A^{-1}D^{-1}$

Solution:

$$DA = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix} \times \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 4 + 1 \times (-1) & 3 \times 0 + 1 \times 2 \\ -2 \times 4 + 2 \times (-1) & -2 \times 0 + 2 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 - 1 & 0 + 2 \\ -8 - 2 & 0 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 2 \\ -10 & 4 \end{bmatrix}$$

As, $(DA)^{-1} = \dfrac{Adj\,DA}{|DA|}$

$$Adj\,DA = \begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$$

$$|DA| = \begin{vmatrix} 11 & 2 \\ -10 & 4 \end{vmatrix} = 11 \times 4 - (-10) \times 2$$

$$= 44 + 20 = 64 \neq 0$$

$$(DA)^{-1} = \frac{\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}}{64} = \begin{bmatrix} \frac{4}{64} & \frac{-2}{64} \\ \frac{10}{64} & \frac{11}{64} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix}$$

$$(DA)^{-1} = \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix} \text{ ---------- (i)}$$

$$A^{-1} = \frac{Adj\,A}{|A|}$$

$$Adj\,A = \begin{bmatrix} 2 & -0 \\ 1 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix} = 4 \times 2 + (-1) \times 0$$

$$= 8 - 0 = 8 \neq 0$$

$$A^{-1} = \frac{\begin{bmatrix} 2 & -0 \\ 1 & 4 \end{bmatrix}}{8} = \begin{bmatrix} \frac{2}{8} & \frac{-0}{8} \\ \frac{1}{8} & \frac{4}{8} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix} \text{ ---------- (a)}$$

$$D^{-1} = \frac{Adj\,D}{|D|}$$

$$Adj\,D = \begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$|D| = \begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix} = 3 \times 2 - (-2) \times 1$$

$$= 6 + 2 = 8 \neq 0$$

$$(D)^{-1} = \frac{\begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}}{8} = \begin{bmatrix} \frac{2}{8} & \frac{-1}{8} \\ \frac{2}{8} & \frac{3}{8} \end{bmatrix}$$

$$D^{-1} = \begin{bmatrix} \frac{1}{4} & \frac{-1}{8} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix} \text{ ---------- (b)}$$

Now by solving R.H.S

$$= A^{-1}D^{-1}$$

$$= \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix} \times \begin{bmatrix} \frac{1}{4} & \frac{-1}{8} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} \times \frac{1}{4} + 0 \times \frac{1}{4} & \frac{1}{4} \times \frac{-1}{8} + 0 \times \frac{3}{8} \\ \frac{1}{8} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} & \frac{1}{8} \times \frac{-1}{8} + \frac{1}{2} \times \frac{3}{8} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} + 0 & -\frac{1}{32} + 0 \\ \frac{1}{32} + \frac{1}{8} & . -\frac{1}{64} + \frac{3}{16} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{1+4}{32} & \frac{-1+12}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix} \text{ ------------ (ii)}$$

From (i) and (ii), it is clear that:

$(DA)^{-1} = A^{-1}D^{-1}$ Hence proved

EXERCISE 1.6

Q1. Use matrices, if possible, to solve the following systems of linear equations by:

(i) the matrix inverse method

(ii) the Cramer's rule.

(i) $2x - 2y = 4$
$3x + 2y = 6$

(ii) $2x + y = 3$
$6x + 5y = 1$

(iii) $4x + 2y = 8$
$3x - y = -1$

(iv) $3x - 2y = -6$
$5x - 2y = -10$

$$(D)^{-1} = \frac{\begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}}{8} = \begin{bmatrix} \frac{2}{8} & \frac{-1}{8} \\ \frac{2}{8} & \frac{3}{8} \end{bmatrix}$$

$$D^{-1} = \begin{bmatrix} \frac{1}{4} & \frac{-1}{8} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix} \quad \text{----------- (b)}$$

Now by solving R.H.S

$$= A^{-1}D^{-1}$$

$$= \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix} \times \begin{bmatrix} \frac{1}{4} & \frac{-1}{8} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} \times \frac{1}{4} + 0 \times \frac{1}{4} & \frac{1}{4} \times \frac{-1}{8} + 0 \times \frac{3}{8} \\ \frac{1}{8} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} & \frac{1}{8} \times \frac{-1}{8} + \frac{1}{2} \times \frac{3}{8} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} + 0 & -\frac{1}{32} + 0 \\ \frac{1}{32} + \frac{1}{8} & -\frac{1}{64} + \frac{3}{16} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{1+4}{32} & \frac{-1+12}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix} \quad \text{------------- (ii)}$$

From (i) and (ii), it is clear that:

$(DA)^{-1} = A^{-1}D^{-1}$ Hence proved

EXERCISE 1.6

Q1. Use matrices, if possible, to solve the following systems of linear equations by:

(i) the matrix inverse method

(ii) the Cramer's rule.

(i) $2x - 2y = 4$
$3x + 2y = 6$

(ii) $2x + y = 3$
$6x + 5y = 1$

(iii) $4x + 2y = 8$
$3x - y = -1$

(iv) $3x - 2y = -6$
$5x - 2y = -10$

(v) $3x - 2y = 4$; $-6x + 4y = 7$ (vi) $4x + y = 9$; $-3x - y = -5$

(vii) $2x - 2y = 4$; $-5x - 2y = -10$ (viii) $3x - 4y = 4$; $x + 2y = 8$

(i) Solution By Matrix Inversion Method:

(i) $2x - 2y = 4$; $3x + 2y = 6$

Solution:

Step 1

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} = 2 \times 2 - 3 \times (-2) = 4 + 6 = 10 \neq 0$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\ M \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 2 \times 4 + 2 \times 6 \\ -3 \times 4 + 2 \times 6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 8 + 12 \\ -12 + 12 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad x = 2, \quad y = 0$

(ii) $2x + y = 3$; $6x + 5y = 1$

Solution:

Step 1

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix} = 2 \times 5 - 1 \times 6 = 10 - 6 = 4 \neq 0$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\, M \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 5 \times 3 + (-1) \times 1 \\ -6 \times 3 + 2 \times 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 14 \\ -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

$$\Rightarrow \quad x = \frac{7}{2}, \quad y = -4$$

(iii) $\mathbf{4x + 2y = 8}$; $\mathbf{3x - y = -1}$

Solution:

Step 1

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Step 2

The coefficient matrix $M = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix} = 4 \times (-1) - 2 \times 3 = -4 - 6 = -10 \neq 0$$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\, M \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$= -\frac{1}{10}\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$= -\frac{1}{10}\begin{bmatrix} (-1) \times 8 + (-2) \times (-1) \\ -3 \times 8 + 4 \times (-1) \end{bmatrix}$$

$$= -\frac{1}{10}\begin{bmatrix} -8 + 2 \\ -24 + -4 \end{bmatrix}$$

$$= -\frac{1}{10}\begin{bmatrix} -6 \\ -28 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ \frac{14}{5} \end{bmatrix}$$

$$\Rightarrow \quad x = \frac{3}{5}, \quad y = \frac{14}{5}$$

(iv) $3x - 2y = -6$; $5x - 2y = -10$

Solution:

Step 1

$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix} = 3\times(-1) - 5 \times(-2)$$
$$= -6 + 10 = 4 \neq 0$$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} -6 \\ -10 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\, M \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$
$$= \frac{1}{4}\begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} -6 \\ -10 \end{bmatrix}$$
$$= \frac{1}{4}\begin{bmatrix} (-2)\times(-6) + (2)\times(-10) \\ (-5)\times(-6) + 3 \times(-10) \end{bmatrix}$$
$$= \frac{1}{4}\begin{bmatrix} 12 - 20 \\ 30 - 30 \end{bmatrix}$$
$$= \frac{1}{4}\begin{bmatrix} -8 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad x = -2, \quad y = 0$

(v) $3x - 2y = 4$; $-6x + 4y = 7$

Solution:

Step 1

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$ is singular because

$$\det M = \begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix} = 3\times 4 - (-2)\times(-6) = 12 - 12 = 0$$

So, M is a singular matrix. Hence the system of linear equations has no solution.

(vi) **4x + y = 9** ; **− 3x − y = − 5**

Solution:

Step 1

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix} = 4 \times (-1) - 3 \times 1$$

$$= -4 + 3 = -1 \neq 0$$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 9 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\, M \begin{bmatrix} 9 \\ 5 \end{bmatrix}$$

$$= \frac{1}{-1} \begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$= -1 \begin{bmatrix} (-1) \times 9 + (-1) \times (-5) \\ 3 \times (9 + 4 \times (-5) \end{bmatrix}$$

$$= -1 \begin{bmatrix} -9 + 5 \\ 27 - 20 \end{bmatrix}$$

$$= -1 \begin{bmatrix} -4 \\ 7 \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$$

$\Rightarrow \quad x = 4, \quad y = -7$

(vii) **2x − 2y = 4** ; **− 5x − 2y = − 10**

Solution:

Step 1

$$\begin{bmatrix} 2 & 2 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 2 & 2 \\ -5 & -2 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 2 & 2 \\ -5 & -2 \end{vmatrix} = 2 \times (-2) - 5 \times 2$$

$$= -4 - 10 = -14 \neq 0$$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\, M \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$= \frac{1}{-14}\begin{bmatrix}-2 & -2\\ 5 & 2\end{bmatrix}\begin{bmatrix}4\\ -10\end{bmatrix}$$

$$= \frac{1}{-14}\begin{bmatrix}(-2)\times 4+2\times(-10)\\ 5\times 4+2\times(-10)\end{bmatrix}$$

$$= \frac{1}{-14}\begin{bmatrix}-8-20\\ 20-20\end{bmatrix}$$

$$= \frac{1}{-14}\begin{bmatrix}-28\\ 0\end{bmatrix} = \begin{bmatrix}2\\ 0\end{bmatrix}$$

$\Rightarrow \quad x = 2, \quad y = 0$

(viii) 3x – 4y = 4 ; x + 2y = 8

Solution:

Step 1

$$\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}4\\ 8\end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix}3 & -4\\ 1 & 2\end{vmatrix} = 3\times 2-(-4)\times 1 = 6+4 = 10 \neq 0$

Step 3

$$\begin{bmatrix}x\\ y\end{bmatrix} = M^{-1}\begin{bmatrix}4\\ 8\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix} = \frac{1}{|M|}Adj\ M\begin{bmatrix}4\\ 8\end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix}2 & 4\\ -1 & 3\end{bmatrix}\begin{bmatrix}4\\ 8\end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix}(2)\times 4+4\times 8\\ (-1)\times 4+3\times 8\end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix}8+32\\ -4+24\end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix}40\\ 20\end{bmatrix} = \begin{bmatrix}4\\ 2\end{bmatrix}$$

$\Rightarrow \quad x = 4, \quad y = 2$

(ii) Solution By Cramer's Rule:

(i) 2x – 2y = 4 ; 3x + 2y = 6

Solution:

$$\begin{bmatrix}2 & -2\\ 3 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}4\\ 6\end{bmatrix}$$

$$A = \begin{bmatrix}2 & -2\\ 3 & 2\end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} = 2 \times 2 - 3 \times (-2)$$
$$= 4 + 6 = 10 \neq 0$$
$$A_x = \begin{bmatrix} 4 & -2 \\ 6 & 2 \end{bmatrix}$$
$$|A_x| = \begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix} = 4 \times 2 - 6 \times (-2)$$
$$= 8 + 12 = 20$$
$$A_y = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$$
$$|A_y| = \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} = 2 \times 6 - 3 \times 4$$
$$= 12 - 12 = 0$$
$$x = \frac{|A_x|}{|A|} = \frac{20}{10} = 2$$
$$y = \frac{|A_y|}{|A|} = \frac{0}{10} = 0$$

So, $x = 2$ **and** $y = 0$

(ii) $\mathbf{2x + y = 3}$; $\mathbf{6x + 5y = 1}$

Solution:

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$
$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$$
$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix} = 2 \times 5 - 6 \times 1$$
$$= 10 - 6 = 4 \neq 0$$
$$A_x = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$$
$$|A_x| = \begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} = 3 \times 5 - 1 \times 1$$
$$= 15 - 1 = 14$$
$$A_y = \begin{bmatrix} 2 & 3 \\ 6 & 1 \end{bmatrix}$$
$$|A_y| = \begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix} = 2 \times 1 - 6 \times 3$$
$$= 2 - 18 = -16$$
$$x = \frac{|A_x|}{|A|} = \frac{14}{4} = \frac{7}{2}$$
$$y = \frac{|A_y|}{|A|} = \frac{-16}{4} = -4$$

So, $x = \frac{7}{2}$ **and** $y = -4$

(iii) $4x + 2y = 8$; $3x - y = -1$

Solution:

$$\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}8\\-1\end{bmatrix}$$

$$A = \begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}$$

$$|A| = \begin{vmatrix}4 & 2\\3 & -1\end{vmatrix} = 4 \times (-1) - 3 \times 2$$

$$= -4 - 6 = -10 \neq 0$$

$$A_x = \begin{bmatrix}8 & 2\\-1 & -1\end{bmatrix}$$

$$|A_x| = \begin{vmatrix}8 & 2\\-1 & -1\end{vmatrix} = 8 \times (-1) - 2 \times (-1)$$

$$= -8 + 2 = -6$$

$$A_y = \begin{bmatrix}4 & 8\\3 & -1\end{bmatrix}$$

$$|A_y| = \begin{vmatrix}4 & 8\\3 & -1\end{vmatrix} = 4 \times (-1) - 3 \times 8$$

$$= -4 - 24 = -28$$

$$x = \frac{|A_x|}{|A|} = \frac{-6}{-10} = \frac{3}{5}$$

$$y = \frac{|A_y|}{|A|} = \frac{-28}{-10} = \frac{14}{5}$$

So, $x = \frac{3}{5}$ and $y = \frac{14}{5}$

(iv) $3x - 2y = -6$; $5x - 2y = -10$

Solution:

$$\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-6\\-10\end{bmatrix}$$

$$A = \begin{bmatrix}3 & -2\\5 & -2\end{bmatrix}$$

$$|A| = \begin{vmatrix}3 & -2\\5 & -2\end{vmatrix} = 3 \times (-2) - 5 \times (-2)$$

$$= -6 + 10 = 4 \neq 0$$

$$A_x = \begin{bmatrix}-6 & -2\\-10 & -2\end{bmatrix}$$

$$|A_x| = \begin{vmatrix}-6 & -2\\-10 & -2\end{vmatrix}$$

$$= (-6) \times (-2) - (-10) \times (-2)$$

$$= 12 - 20 = -8$$

$$A_y = \begin{bmatrix}3 & -6\\5 & -10\end{bmatrix}$$

$$|A_y| = \begin{vmatrix}3 & -6\\5 & -10\end{vmatrix}$$

$$= 3\times(-10) - 5\times(-6)$$
$$= -30 + 30 = 0$$
$$x = \frac{|A_x|}{|A|} = \frac{-8}{4} = -2$$
$$y = \frac{|A_y|}{|A|} = \frac{0}{4} = 0$$

So, $x = -2$ and $y = 0$

(v) $3x - 2y = 4$; $-6x + 4y = 7$

Solution:

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$
$$A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$$
$$|A| = \begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix} = 3\times 4 - (-6)\times(-2)$$
$$= 12 - 12 = 0$$

Since it is singular matrix therefore solution is not possible.

So, x and y are not possible in this case.

(vi) $4x + y = 9$; $-3x - y = -5$

Solution:

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$
$$A = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$$
$$|A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix} = 4\times(-1) - (-3)\times 1$$
$$= -4 + 3 = -1$$
$$A_x = \begin{bmatrix} 9 & 1 \\ -5 & -1 \end{bmatrix}$$
$$|A_x| = \begin{vmatrix} 9 & 1 \\ -5 & -1 \end{vmatrix} = 9\times(-1) - (-5)\times 1$$
$$= -9 + 5 = -4$$
$$A_y = \begin{bmatrix} 4 & 9 \\ -3 & -5 \end{bmatrix}$$
$$|A_y| = \begin{vmatrix} 4 & 9 \\ -3 & -5 \end{vmatrix} = 4\times(-5) - (-3)\times 9$$
$$= -20 + 27 = 7$$
$$x = \frac{|A_x|}{|A|} = \frac{-4}{-1} = 4$$
$$y = \frac{|A_y|}{|A|} = \frac{7}{-1} = -7$$

So, $x = 4$ and $y = -7$

(vii) $2x - 2y = 4$; $-5x - 2y = -10$

Solution:

$$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix} = 2 \times (-2) - (-5) \times (-2)$$

$$= -4 - 10 = -14 \neq 0$$

$$A_x = \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 4 & 2 \\ -10 & -2 \end{vmatrix} = 4 \times (-2) - (-10) \times 2$$

$$|A_x| = -8 - 20 = -28$$

$$A_y = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 2 & 4 \\ -5 & -10 \end{vmatrix} = 2 \times (-10) - (-5) \times 4$$

$$|A_y| = -20 + 20 = 0$$

$$x = \frac{|A_x|}{|A|} = \frac{-28}{-14} = 2$$

$$y = \frac{|A_y|}{|A|} = \frac{0}{-14} = 0$$

So, $x = 2$ and $y = 0$

(viii) $3x - 4y = 4$; $x + 2y = 8$

Solution:

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix} = 3 \times 2 - 1 \times (-4)$$

$$= 6 + 4 = 10 \neq 0$$

$$A_x = \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix} = 4 \times 2 - 8 \times (-4)$$

$$|A_x| = 8 - 32 = 40$$

$$A_y = \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix} = 3 \times 8 - 1 \times 4$$

$$|A_y| = 24 - 4 = 20$$

$$x = \frac{|A_x|}{|A|} = \frac{40}{10} = 4$$

$$y = \frac{|A_y|}{|A|} = \frac{20}{10} = 2$$

So, $x = 4$ **and** $y = 2$

Q2. The length of a rectangle is 4 times its width. The perimeter of the rectangle is 150 cm. Find the dimensions of the rectangle.

Solution:

(i) Method 1: Matrix Inversion Method:

Let length of rectangle is x cm and width of rectangle is y cm.

According to given condition

x = 4y

or x – 4y = 0 -------- (i)

Perimeter = 150 cm

Perimeter = 2(x + y) = 150

or x + y = 75 ------- (ii)

By solving (i) and (ii), we get

$$\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

The coefficient matrix

$M = \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 1 & -4 \\ 1 & 1 \end{vmatrix} = 1 \times 1 - 1 \times (-4) = 1 + 4 = 5 \neq 0$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\ M \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 1 \times 0 + 4 \times 75 \\ (-1) \times 0 + 1 \times 75 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 0 + 300 \\ 0 + 75 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 300 \\ 75 \end{bmatrix} = \begin{bmatrix} 60 \\ 15 \end{bmatrix}$$

$\Rightarrow$ $x = 60, \quad y = 15$

So length = x = 60 cm

width = y = 15 cm.

(ii) (i) Method 2: By Cramer's rule:

$x - 4y = 4$; $x + y = 8$

$\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & -4 \\ 1 & 1 \end{vmatrix} = 1 \times 1 - (-4) \times 1$

$= 1 + 4 = 5 \neq 0$

$A_x = \begin{bmatrix} 0 & -4 \\ 75 & 1 \end{bmatrix}$

$|A_x| = \begin{vmatrix} 0 & -4 \\ 75 & 1 \end{vmatrix} = 0 \times 1 - (-4) \times (75)$

$|A_x| = 0 + 300 = 300$

$A_y = \begin{bmatrix} 1 & 0 \\ 1 & 75 \end{bmatrix}$

$|A_y| = \begin{vmatrix} 1 & 0 \\ 1 & 75 \end{vmatrix} = 1 \times 75 - 0 \times 1$

$|A_y| = 75 - 0 = 75$

$x = \frac{|A_x|}{|A|} = \frac{300}{5} = 60$

$y = \frac{|A_y|}{|A|} = \frac{75}{5} = 15$

$\Rightarrow$ $x = 60, \quad y = 15$

So length = x = 60 cm ; width = y = 15 cm.

Q3. Two sides of a rectangle differ by 3.5 cm. Find the dimensions of the rectangle if its perimeter is 67 cm.

Solution:

(i) Method 1: Matrix Inversion Method:

Let the length of the rectangle is x cm and its width is y cm.

According to given condition

$\therefore$ $x - y = 3.5$

and $2x + 2y = 67$

or $10x - 10y = 35$

and $2x + 2y = 67$

$\begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35 \\ 67 \end{bmatrix}$

The coefficient matrix

$M = \begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 10 & -10 \\ 2 & 2 \end{vmatrix} = 3\times(-1) - 5 \times (-2)$$

$$= -6 + 10 = 4 \neq 0$$

So, M is a singular matrix

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 35 \\ 67 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\ M \begin{bmatrix} 35 \\ 67 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 2 & 10 \\ -2 & 10 \end{bmatrix}\begin{bmatrix} 35 \\ 67 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 2\times 35 + 10 \times 67 \\ -2 \times 35 + 10 \times 67 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 70 + 670 \\ -70 + 670 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 740 \\ 600 \end{bmatrix} = \begin{bmatrix} 18.5 \\ 15 \end{bmatrix}$$

$\Longrightarrow \quad x = 18.5, y = 15$

So the length is 18.5 cm and 15 cm.

(ii) Method 2: By Cramer's rule:

$10x - 10y = 35 \quad ; \quad 2x + 2y = 67$

$$\begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35 \\ 67 \end{bmatrix}$$

$$A = \begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 10 & -10 \\ 2 & 2 \end{vmatrix} = 10 \times 2 - (-10) \times 2$$

$$= 20 + 20 = 40 \neq 0$$

$$A_x = \begin{bmatrix} 35 & -10 \\ 67 & 2 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 35 & -10 \\ 67 & 2 \end{vmatrix}$$

$$= 35 \times 2 - (-10) \times (67)$$

$$|A_x| = 70 + 670 = 740$$

$$A_y = \begin{bmatrix} 10 & 35 \\ 2 & 67 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 10 & 35 \\ 2 & 67 \end{vmatrix} = 10 \times 67 - 35 \times 2$$

$$|A_y| = 670 - 70 = 600$$

$$x = \frac{|A_x|}{|A|} = \frac{740}{40} = 18.5$$

$$y = \frac{|A_y|}{|A|} = \frac{600}{40} = 15$$

$\Longrightarrow \quad x = 18.5, y = 15$

So the length is 18.5 cm and 15 cm.

Q4. The third angle of an isosceles triangle is 16° less than the sum of the two equal angles. Find three angles of the triangle.

Solution:

(i) Method 1: Matrix Inversion Method:

Let the each equal angle be x° cm and the third angle be y° cm

∴ $2x - 16 = y$

and $2x + y = 180°$

or $2x - y = 16$

and $2x + y = 180$

or $\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$

The coefficient matrix M = $\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} = 2 \times 1 - (-1) \times 2 = 2 + 2 = 4 \neq 0$

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\, M \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 1 \times 16 + 1 \times 180 \\ -2 \times 16 + 2 \times 180 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 16 + 180 \\ -32 + 360 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 196 \\ 328 \end{bmatrix} = \begin{bmatrix} 49 \\ 82 \end{bmatrix}$$

$\Longrightarrow$ $x = 49, \quad y = 82$

$x + y + z = 180^0$

$49^0 + 82^0 + z = 180^0$

$z = 180^0 - 49^0 - 82^0 = 49^0$

So the angles are 49°, 49°, 82°

(ii) Method 2: By Cramer's rule:

$2x - y = 16 \quad ; \quad 2x + y = 180$

$$\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} = 2 \times 1 - (-1) \times 2$$

$$= 2 + 2 = 4 \neq 0$$

$$A_x = \begin{bmatrix} 16 & -1 \\ 80 & 1 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 16 & -1 \\ 180 & 1 \end{vmatrix}$$

$$= 16 \times 1 - (-1) \times (180)$$

$$|A_x| = 16 + 180 = 196$$

$$A_y = \begin{bmatrix} 2 & 16 \\ 2 & 80 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 2 & 16 \\ 2 & 80 \end{vmatrix} = 2 \times 180 - 16 \times 2$$

$$|A_y| = 360 - 32 = 328$$

$$x = \frac{|A_x|}{|A|} = \frac{196}{4} = 49$$

$$y = \frac{|A_y|}{|A|} = \frac{328}{4} = 82$$

$\Longrightarrow$ $x = 49, \quad y = 82$

$x + y + z = 180^0$

$49^0 + 82^0 + z = 180^0$

$z = 180^0 - 49^0 - 82^0 = 49^0$

So the angles are 49°, 49°, 82°

Q5. One acute angle of a right triangle is 12° more than twice the other acute angle. Find the acute angles of the right triangle.

Solution:

(i) Method 1: Matrix Inversion Method:

Let the two angles be x° cm and y° cm.

$\therefore$ $2x + 12 = y$

or $2x - y = -12$

and $x + y = 90$ (sum of angles)

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

The coefficient matrix $M = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} = 2 \times 1 - (-1) \times 1 = 2 + 1 = 3 \neq 0$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\ M \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 \times (-12) + 1 \times 90 \\ -1 \times (-12) + 2 \times 90 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} -12 + 90 \\ 12 + 180 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 78 \\ 192 \end{bmatrix} = \begin{bmatrix} 26 \\ 64 \end{bmatrix}$$

$\Rightarrow$ $x = 26, y = 64$

So the angles are 26°, 64°.

(ii) Method 2: By Cramer's rule:

$2x - y = -12 \quad ; \quad x + y = 90$

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 \times 1 - (-1) \times 1$$

$$= 2 + 1 = 3 \neq 0$$

$$A_x = \begin{bmatrix} -12 & -1 \\ 90 & 1 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} -12 & -1 \\ 90 & 1 \end{vmatrix}$$

$$= (-12) \times 1 - (-1) \times (90)$$

$$|A_x| = -12 + 90 = 78$$

$$A_y = \begin{bmatrix} 90 & 12 \\ -1 & 2 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 90 & 12 \\ -1 & 2 \end{vmatrix}$$

$$= 90 \times 2 - 12 \times (-1)$$

$$|A_y| = 180 + 12 = 192$$

$$x = \frac{|A_x|}{|A|} = \frac{78}{3} = 26$$

$$y = \frac{|A_y|}{|A|} = \frac{192}{3} = 64$$

$\Rightarrow$ $x = 26, y = 64$

So the angles are 26°, 64°.

Q6. Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

Solution:

(i) Method 1: Matrix Inversion Method:

Let the speeds of two cars be

x km/h and x km/h

$x - y = 6$ ------------------- (i)

Distance covered in $4\frac{1}{2}$ hours = 600 – 123 = 477 km

$$4\frac{1}{2}x + 4\frac{1}{2}y = 477$$

$$\frac{9}{2}x + \frac{9}{2}y = 477$$

or $$\frac{x}{2} + \frac{y}{2} = 53$$

or $$x + y = 106$$ ---------------- (ii)

The two equations are (i) and (ii)

$$\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}6\\106\end{bmatrix}$$

The coefficient matrix M = $\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix}1 & -1\\1 & 1\end{vmatrix} = 1 \times 1 - 1 \times (-1) = 1 + 1 = 2 \neq 0$$

$$\begin{bmatrix}x\\y\end{bmatrix} = M^{-1}\begin{bmatrix}6\\106\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix} = \frac{1}{|M|}Adj\,M\begin{bmatrix}6\\106\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}\begin{bmatrix}6\\106\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}1 \times 6 + 1 \times 106\\-1 \times 6 + 1 \times 106\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}6 + 106\\-6 + 106\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}112\\100\end{bmatrix} = \begin{bmatrix}56\\50\end{bmatrix}$$

$\Rightarrow$ $x = 56, y = 50$

$\therefore$ The speeds of the two cars are 56 km/h and 50 km/h

(ii) Method 2: By Cramer's rule:

$x - y = 6$; $x + y = 106$

$$\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}6\\106\end{bmatrix}$$

$$A = \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$$

$$|A| = \begin{vmatrix}1 & -1\\1 & 1\end{vmatrix} = 1 \times 1 - (-1) \times 1$$

$$= 1 + 1 = 2 \neq 0$$

$$A_x = \begin{bmatrix}6 & -1\\106 & 1\end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 6 & -1 \\ 106 & 1 \end{vmatrix}$$

$$= 6 \times 1 - (-1) \times (106)$$

$$|A_x| = 6 + 106 = 112$$

$$A_y = \begin{bmatrix} 1 & 6 \\ 1 & 106 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 1 & 6 \\ 1 & 106 \end{vmatrix} = 1 \times 106 - 6 \times 1$$

$$|A_y| = 106 - 6 = 100$$

$$x = \frac{|A_x|}{|A|} = \frac{112}{2} = 56$$

$$y = \frac{|A_y|}{|A|} = \frac{100}{2} = 50$$

$\Rightarrow$ $x = 56, y = 50$

The speeds of the two cars are 56 km/h and 50 km/h

REVIEW EXERCISE 1

Q1. Select the correct answer in each of the following.

(i) The order of matrix [2 1] is.........

(a) 2-by-1 **(b)** 1-by-2
(c) 1-by-1 **(d)** 2-by-2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ **is called.........matrix.**

(a) zero **(b)** unit
(c) scalar **(d)** singular

(iii) Which is order of a square matrix..........

(a) 2-by-2 **(b)** l-by-2
(c) 2-by-1 **(d)** 3-by-2

(iv) Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ **is.........**

(a) 3-by-2 **(b)** 2-by-3
(c) 1-by-3 **(d)** 3-by-1

(v) Ad joint of $\begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix}$ **is...........**

(a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$ **(d)** $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

$$|A_x| = \begin{vmatrix} 6 & -1 \\ 106 & 1 \end{vmatrix}$$

$$= 6 \times 1 - (-1) \times (106)$$

$$|A_x| = 6 + 106 = 112$$

$$A_y = \begin{bmatrix} 1 & 6 \\ 1 & 106 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 1 & 6 \\ 1 & 106 \end{vmatrix} = 1 \times 106 - 6 \times 1$$

$$|A_y| = 106 - 6 = 100$$

$$x = \frac{|A_x|}{|A|} = \frac{112}{2} = 56$$

$$y = \frac{|A_y|}{|A|} = \frac{100}{2} = 50$$

$\Rightarrow$ $x = 56, y = 50$

The speeds of the two cars are 56 km/h and 50 km/h

REVIEW EXERCISE 1

Q1. Select the correct answer in each of the following.

(i) The order of matrix [2 1] is.........

(a) 2-by-1 **(b)** 1-by-2
(c) 1-by-1 **(d)** 2-by-2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ **is called.........matrix.**

(a) zero **(b)** unit
(c) scalar **(d)** singular

(iii) Which is order of a square matrix..........

(a) 2-by-2 **(b)** l-by-2
(c) 2-by-1 **(d)** 3-by-2

(iv) Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ **is.........**

(a) 3-by-2 **(b)** 2-by-3
(c) 1-by-3 **(d)** 3-by-1

(v) Ad joint of $\begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix}$ **is...........**

(a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$ **(d)** $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

(vi) **Product of** $[x \quad y]\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **is.........**

(a) $[2x + y]$ **(b)** $[x - 2y]$
(c) $[2x - y]$ **(d)** $[x + 2y]$

(vii) **If** $\begin{bmatrix} 2 & 6 \\ 3 & x \end{bmatrix}$ **= 0, then x is equal to...a = ...**

(a) 9 **(b)** -6
(c) 6 **(d)** -9

(viii) **If** $X + \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, **then** X **is equal to.........**

(a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ **(d)** $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

Answers:

(i) b	**(ii) c**	**(iii) a**	**(iv) b**
(v) a	**(vi) c**	**(vii) a**	**(viii) d**

Q2. **Complete the following:**

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called.........matrix.

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called.........matrix.

(iii) Additive inverse of $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ is....

(iv) In matrix multiplication, in general. AB...≠......BA.

(v) Matrix A + B may be found if order of A and B is........

(vi) A matrix is called........matrix if number of rows and columns are equal.

Answers:

(i) Null	**(ii) Unit**	**(iii)** $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$
(iv) ≠	**(v) Same**	**(vi) Square**

Q3. **If** $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$, **then find a and b.**

Solution:

By comparing the corresponding elements, we get

$$a + 3 = -3$$
$$a = -3 - 3 = -6$$
$$a = -6$$
$$b - 1 = 2$$

$$b = 2 + 1$$
$$b = 3$$

Q4. **If** $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$, **then find the following.**

(i) $2A + 3B$ **(ii)** $-3A + 2B$

(iii) $-3(A + 2B)$ **(iv)** $\frac{2}{3}(2A - 3B)$

Solution:

(i) $2A + 3B$

$$= 2 \times \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 3 \times \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 2 & 2 \times 3 \\ 2 \times 1 & 2 \times 0 \end{bmatrix} + \begin{bmatrix} 3 \times 5 & 3 \times (-4) \\ 3 \times (-2) & 3 \times (-1) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 15 & 6 - 12 \\ 2 - 6 & 0 - 3 \end{bmatrix}$$

$$2A + 3B = \begin{bmatrix} 19 & -6 \\ -4 & -3 \end{bmatrix}$$

Solution:

(ii) $-3A + 2B$

$$= -3 \times \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2 \times \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \times 2 & -3 \times 3 \\ -3 \times 1 & -3 \times 0 \end{bmatrix} + \begin{bmatrix} 2 \times 5 & 2 \times (-4) \\ 2 \times (-2) & 2 \times (-1) \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -9 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6 + 10 & -9 + (-8) \\ -3 + (-4) & 0 + (-2) \end{bmatrix}$$

$$-3A + 2B = \begin{bmatrix} 4 & -17 \\ -7 & -2 \end{bmatrix}$$

(iii) $-3(A + 2B)$

$$= -3\left(\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2 \times \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}\right)$$

$$= -3\left(\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 \times 5 & 2 \times -4 \\ 2 \times -2 & 2 \times -1 \end{bmatrix}\right)$$

$$= -3\left(\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}\right)$$

$$= -3\begin{bmatrix} 2 + 10 & 3 - 8 \\ 1 - 4 & 0 - 2 \end{bmatrix}$$

$$= -3\begin{bmatrix} 12 & -5 \\ -3 & -2 \end{bmatrix}$$

$$= -\begin{bmatrix} 3\times 12 & 3\times(-5) \\ 3\times(-3) & 3\times(-2) \end{bmatrix}$$

$$= -\begin{bmatrix} 36 & -15 \\ -9 & -6 \end{bmatrix}$$

$$\mathbf{-3(A+2B)} = \begin{bmatrix} -36 & 15 \\ 9 & 6 \end{bmatrix}$$

(iv) $\frac{2}{3}(2A - 3B)$

$$= \frac{2}{3}\left(2 \times \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} - 3 \times \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}\right)$$

$$= \frac{2}{3}\left(\begin{bmatrix} 2\times 2 & 2\times 3 \\ 2\times 1 & 2\times 0 \end{bmatrix} - \begin{bmatrix} 3\times 5 & 3\times(-4) \\ 3\times(-2) & 3\times(-1) \end{bmatrix}\right)$$

$$= \frac{2}{3}\left(\begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 4-15 & 6+12 \\ 2+6 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 18 \\ 8 & 3 \end{bmatrix}$$

$$\frac{2}{3}(2A - 3B) = \begin{bmatrix} -\frac{22}{3} & 12 \\ \frac{16}{3} & 2 \end{bmatrix}$$

Q5. Find the value of X, if $\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} + X = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}$.

Solution:

$$\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} + X = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-2 & -2-1 \\ -1-3 & -2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$$

Q6. If $A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$, then prove that

(i) $AB \neq BA$ (ii) $A(BC) = (AB)C$

Solution:

(i) AB $= \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 0\times(-3)+1\times 5 & 0\times 4+1\times(-2) \\ 2\times(-3)+(-3)\times 5 & 2\times 4+(-3)\times(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 0+5 & 0-2 \\ -6-15 & 8+6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ -21 & 14 \end{bmatrix} \quad \text{----------- (i)}$$

$$BA = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} (-3)\times 0 + 4 \times 2 & (-3)\times 1 + 4 \times (-3) \\ 5 \times 0 + (-2) \times 2 & 5 \times 1 + (-2) \times (-3) \end{bmatrix}$$

$$= \begin{bmatrix} 0+8 & -3-12 \\ 0-4 & 5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -15 \\ -4 & 11 \end{bmatrix} \quad \text{---------- (ii)}$$

From (i) and (ii) it is clear that $AB \neq BA$.

(ii) A (BC) = (AB)C

Solution:

Solution is not possible because matrix C is not given.

Q7. If $A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$, then verify that

(i) $(AB)^t = B^t A^t$ (ii) $(AB)^{-1} = B^{-1} A^{-1}$

Solution:

(i) $(AB)^t = B^t A^t$

Solution:

$$A^t = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}^t = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}^t = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 2 + 2 \times (-3) & 3 \times 4 + 2 \times (-5) \\ 1 \times 2 + (-1) \times (-3) & 1 \times 4 + (-1) \times (-5) \end{bmatrix}$$

$$= \begin{bmatrix} 6-6 & 12-10 \\ 2+3 & 4+5 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}$$

$$(AB)^t = \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix} \quad \text{--------------- (i)}$$

$$B^tA^t = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 3 + (-3) \times 2 & 2 \times 1 + (-3) \times (-1) \\ 4 \times 3 + (-5) \times 2 & 4 \times 1 + (-5) \times (-1) \end{bmatrix}$$

$$= \begin{bmatrix} 6-6 & 2+3 \\ 12-10 & 4+5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix} \quad \text{-------------- (ii)}$$

From (i) and (ii) it is clear that $(AB)^t = B^t A^t$

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

Solution:

$$A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$$

$$|A| = 3 \times (-1) - 1 \times 2 = -3 - 2 = -5 \neq 0$$

$$A^{-1} = \frac{Adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}}{-5} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{1}{5} & -\frac{3}{5} \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$$

$$|B| = 2 \times (-5) - 4 \times (-3) = -10 + 12 = 2 \neq 0$$

$$B^{-1} = \frac{Adj\ B}{|B|}$$

$$= \frac{1}{2}\begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{5}{2} & -2 \\ \frac{3}{2} & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 2 + 2 \times (-3) & 3 \times 4 + 2 \times (-5) \\ 1 \times 2 + (-1) \times (-3) & 1 \times 4 + (-1) \times (-5) \end{bmatrix}$$

$$= \begin{bmatrix} 6 - 6 & 12 - 10 \\ 2 + 3 & 4 + 5 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}$$

$$|AB| = 0 \times 9 - 2 \times 5 = 0 - 10 = -10 \neq 0$$

$$(AB)^{-1} = \frac{Adj\ AB}{|AB|}$$

$$= \frac{1}{10}\begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-9}{10} & \frac{1}{5} \\ \frac{1}{2} & 0 \end{bmatrix} \quad \text{------------ (i)}$$

Now solving $B^{-1}A^{-1}$

$$= \begin{bmatrix} -\frac{5}{2} & -2 \\ \frac{3}{2} & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{1}{5} & -\frac{3}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{-5}{2}\right) \times \frac{1}{5} + (-2) \times \frac{1}{5} & -\frac{5}{2} \times \frac{2}{5} + (-2) \times (-\frac{3}{5}) \\ \frac{3}{2} \times \frac{1}{5} + 1 \times \frac{1}{5} & \frac{3}{2} \times \frac{2}{5} + 1 \times (-\frac{3}{5}) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2}-\frac{2}{5} & -1+\frac{6}{5} \\ \frac{3}{2}+\frac{1}{5} & \frac{3}{5}-\frac{3}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-5-4}{10} & \frac{-5+6}{5} \\ \frac{3+2}{10} & \frac{3-3}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-9}{10} & \frac{1}{5} \\ \frac{1}{2} & 0 \end{bmatrix} \text{ ——————— (ii)}$$

From (i) and (ii) it is clear that $(AB)^{-1} = B^{-1}A^{-1}$